Journal of Orthodontics/Vol. 28/2001/143–149

An Accurate Three-dimensional Cephalometric System: a Solution for the Correction of Cephalic Malpositioning

YOSHIHIDE MORI, D.D.S., PH.D.,

TAKAHIRO MIYAJIMA, D.D.S., PH.D.,

KATSUHIRO MINAMI, D.D.S., PH.D.,

MASAYOSHI SAKUDA, D.D.S., PH.D.

2nd Department of Oral and Maxillofacial Surgery, Faculty of Dentistry, Osaka University, 1–8, Yamadaoka, Suita, Osaka 565-0871, Japan

Abstract. *In recent years, methods have been developed that calculate three-dimensional (3D) co-ordinates of orthodontic landmarks from lateral and frontal cephalograms. However, precise measurement has been impossible with these methods because, although they corrected the magnification of the image, they did not correct 3D cephalic malpositioning that occurs during the measurement of human subjects. In this study, we developed a 3D cephalometric system that corrected not only for magnification of the image, but also 3D cephalic malpositioning during cephalogram exposure.*

Magnification of the image was corrected for first. Cephalic revolution was then sequentially corrected and divided into elements of x-, y-, and z-axes. The origin was parallelly translated to the mid-point of bilateral porion. In order to examine the accuracy of this system, seven human dry skulls were measured.

The accuracy unaffected by the cephalic revolution in any direction and standard errors was within 0·8 mm in any orthodontic landmarks. It was suggested that this measurement system would have sufficient accuracy for clinical application.

The results indicated that precise cephalometric measurement was possible with this system and it was suggested that its clinical application would be possible.

Index words: Three-dimensional analysis, Cephalometry, Stereograms, Mathematical Correction.

Introduction

A cephalogram assists orthodontic diagnosis and treatment planning. A variety of methods have been established for the analysis of lateral cephalograms (Tweed, 1946; Downs, 1948; Graber, 1952; Ricketts, 1966). However, frontal cephalograms have been limited in their use to the examination of facial asymmetry. This is largely due to distortion, mainly around the ear-rod, caused by cephalic revolution. In recent years, methods have been developed by which one can calculate three-dimensional (3D) co-ordinates of orthodontic landmarks from lateral and frontal cephalograms, for the quantitative evaluation of dento-facial deformities including facial asymmetry (Savara, 1965; Yamazaki *et al*., 1981; Grayson *et al*., 1988; Tasman *et al*., 1989; Bookstein et al., 1991; Spolyar *et al*., 1993). In comparison to computed tomography (CT), the cephalogram has advantages of low levels of exposure to radioactivity and no requirements for special equipment. However, if cephalic malpositioning and magnification of the images are not corrected for, precise measurements cannot be expected. In previous reports, accuracy has been examined by measuring human dry skulls. The correction of 3D cephalic malpositioning, which occurs during exposing cephalograms of human subjects, has not been described.

In this study, a 3D cephalometric system has been developed, which corrects not only magnification of the image, but also cephalic malpositioning. The accuracy of this system was evaluated by measuring dry skulls.

Methods

Development of the system

Composition of the system (Figure 1). Lateral and frontal cephalograms were exposed and traced respectively. Following input into an image processor (NEXUS6800, Kashiwagi Laboratory Co., Tokyo, Japan) mediated by a Charge Coupled Device (CCD) camera (DXC-930, Sony, Tokyo, Japan), landmarks were observed on a monitor, and located manually using a digitizer. The programme, which ran on a personal computer (PC-9801 As21, Nippon Electric Co., Tokyo, Japan) was developed in a BASIC language (N88-BASIC, Nippon Electric Co., Tokyo, Japan). Three-dimensional co-ordinates of landmarks were calculated from the data by this program.

Methodology

The 3D cephalometric system calculates 3D co-ordinates of landmarks on an exposed subject, while correcting for magnification of the image using two-dimensional co-ordinate landmarks on lateral and frontal cephalograms. This is

0301-228X/01/020000+00$02.00

based on identical landmarks location on each cephalogram.

In this study, we have devised a new methodology and constructed a measurement system that corrects for cephalic revolution in frontal and horizontal planes, in addition to correcting for cephalic revolution in the sagital plane as described by Yamazaki (1981).

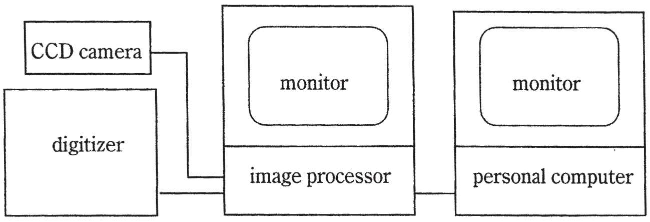


FIG 1 Construction of a computerized cephalometric system.

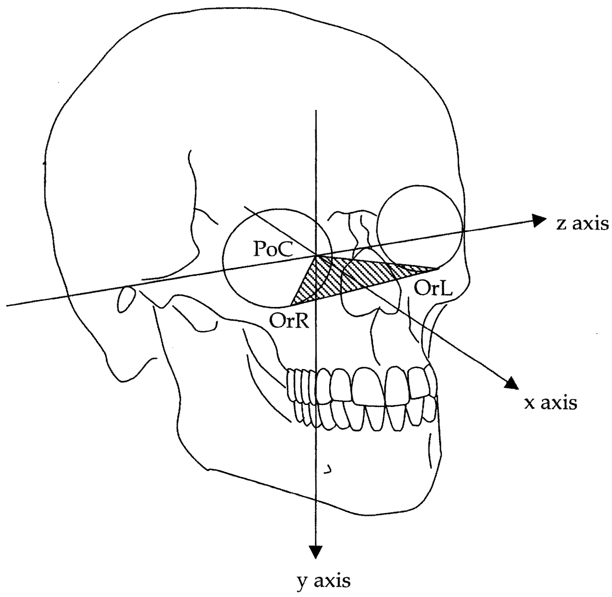


FIG 2 The standard coordinate system. The standard coordinate system is decided by PoC, OrR and OrL.

Establishment of the co-ordinate system

The co-ordinate system used in this study was established as follows (Figure 2). The plane determined by the midpoint (PoC) of bilateral porion (right side: PoR; left side: PoL) and bilateral orbita (right side: OrR; left side: OrL), is defined as the standard plane. The right hand co-ordinate system is decided by PoC fixed as the origin, by the straight line, which passes through PoC and OrC, as the x-axis (posteroanterior direction as positive), by the straight line, which passes through the origin and vertical to the standard plane as y-axis (downward as positive), and by the straight line, which is vertical to x- and y-axis as the z-axis (left direction as positive). This co-ordinate system is defined as the standard co-ordinate system.

Initial processing: correction of magnification of the image. The positions of the tube, the exposed subject and the film were fixed (Figure 3). The cephalogram co-ordinate system was determined by the mid-point of bilateral ear-rods as the origin, by the central line of the X-ray beam of the frontal cephalogram as the x-axis (posteroanterior direction as positive), by the central line of the X-ray beam of the lateral cephalogram as the z-axis (left direction as positive), by the straight line that is vertical to x-and z-axes as the y-axis (downward as positive).

Three-dimensional co-ordinates of landmarks in the cephalogram co-ordinate system were calculated by the correction of magnification of the image. The co-ordinate of an optional point (point A) on an exposed subject in the cephalogram co-ordinate system is defined as (x_0, y_0, z_0). The co-ordinate of point A on the lateral cephalogram is defined as (x_L, y_L), on the frontal cephalogram as (y_{PA}, z_{PA}). The distance between the mid-saggital plane, which is decided by bilateral ear-rods and point A is defined as K (right direction as positive).

The co-ordinate (x_0, y_0), which is corrected for the magnification of the image, is calculated by following numerical formulae:

$$x_0 = x_L \times (1500 + K)/1650 \quad (1)$$

$$y_0 = y_L \times (1500 + K)/1650 \quad (2)$$

y_{PA} is calculated by the following numerical formula:

$$y_{PA} = y_0 \times 1650/(1500 + x_0) \quad (3)$$

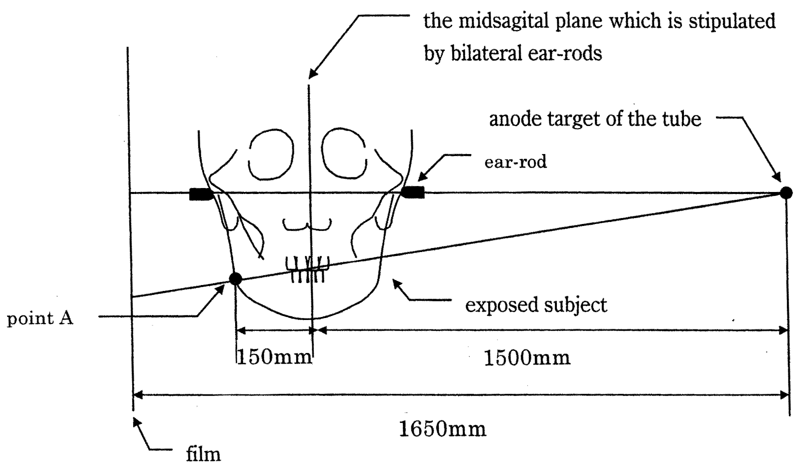


FIG 3 Correction of magnification of the image. Magnification of image at point A is 1650/(1500+k).

A guide line $y = y_{PA}$ is indicated on the frontal cephalogram on the monitor and z_{PA} is detected with reference to this line. Z_0 is calculated by the following numerical formula:

$$z_0 = z_{PA} \times (1500 + x_0)/1650 \quad (4)$$

In order to calculate K for each landmark, an initial value $K0$ (Table 1) is substituted for K, and z_0 is calculated from numerical formulae above. Here, K is nothing else but $-z_0$. In addition, $K0$ is referred to as the average value of Japanese human dry skulls.

By calculating numerical formulae 1_4 again using the value K, correction for the magnification of the image is completed and the co-ordinate (x_0, y_0, z_0) of point A on the exposed subject is obtained.

Second processing: method A (correction for cephalic revolution around the z*-axis).* Cephalic revolution around the z-axis or around the ear-rod often occurs. Yamazaki's correction method uses nasion as a reference point to calculate the revolution angle around the z-axis. However, the detection of nasion on the frontal cephalogram of a human subject is difficult. For this reason, our method adopts the use of upper incisal point (U1), which can be clearly detected from both lateral and frontal cephalograms as the reference. While Yamazaki's method adopted the cephalogram co-ordinate system, our method uses the above-mentioned standard co-ordinate system.

1. Calculation of cephalic revolution angle around the z*-axis, θ_S (Figure 4).* x And y co-ordinate of U1 on an exposed subject are defined as $x_0(U1)$ and $y_0(U1)$, and the y co-ordinate of U1 on the frontal cephalogram as $y_{PA}(U1)$. The revolution angle around z-axis, θ_S (clockwise direction as positive), is summarized by the following numerical formula:

$$\theta_S = \mathrm{Arctan}(y_{PA}) \times \{1650a^2 + 1500 \times [1650^2a^2 - 1500^2y_{PA}(U1)^2 + y_{PA}(U1)^2a^2]^{1/2}\}/[1650^2a^2 - 1500^2y_{PA}(U1)^2] - \mathrm{Arctan}[y_0(U1)/x_0(U1)] \quad (5)$$

However,

$$a = [x_0(U1)^2 + y_0(U1)^2]^{1/2}$$

TABLE 1 *Measured landmarks and their initial K0 values*

Landmark	K0
PoR	60
PoL	–60
OrR	40
OrL	–40
N	0
ANS	0
A	0
B	0
Pog	0
Me	0
GoR	50
GoL	–50

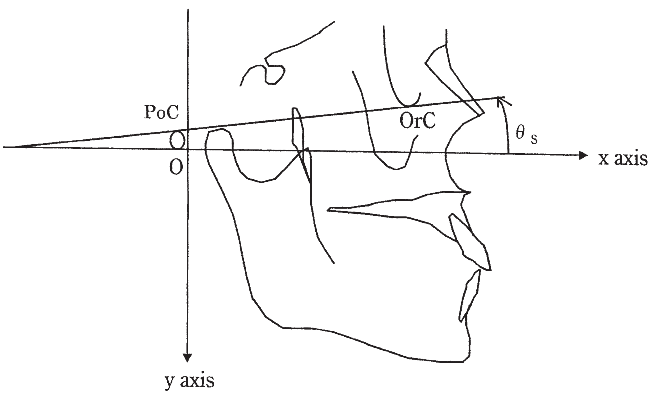


FIG 4 Cephalic revolution angle around the z axis (θS). θS is the angle between the x axis of the cephalogram coordinate system and the straight line which passes through PoC and OrC on the lateral cephalogram.

2. Calculation of the z co-ordinate on the frontal cephalogram. y_{PA} of point A is calculated using numerical formulae 1, 2, 3, and 5 and summarized by the following numerical formula:

$$y_{PA} = (y_0\cos\theta_S + x_0\sin\theta_S) \times 1650/(x_0\cos\theta_S - y_0\sin\theta_S + 1500) \quad (6)$$

A guide line $y = y_{PA}$ is indicated on the frontal cephalogram on a monitor and z_{PA} is detected with reference to this line.

3. Correction for magnification of the z *co-ordinate.* z_0 is calculated using numerical formula 4 and summarized by the following numerical formula:

$$z_0 = z_{PA} \times (1500 + x_0\cos\theta_S - y_0\sin\theta_S)/1650 \quad (7)$$

4. Parallel translation of the origin. While the origin in the standard co-ordinate system is PoC, in order to minimize the influence of cephalic revolution around the y-axis on accuracy, OrC is used as a reference point for parallel translation for the z co-ordinate.

OrC on the frontal cephalogram is decided by detecting the intersection point of the straight line passing bilateral Or and the straight line passing the midpoint (PmC) of the most inside point of the bilateral orbita (right side: PmR, left side: PmL) and vertical to the straight line passing bilateral Or (Figure 5). $z_0(OrC)$ is calculated by substituting $z_{PA}(OrC)$, which is the z co-ordinate on the frontal cephalogram, into numerical formula 7. The co-ordinate (X_0, Y_0, Z_0) of point A on the standard co-ordinate system is calculated by the following numerical formula:

$$\begin{pmatrix} X_0 \\ Y_0 \\ Z_0 \end{pmatrix} = \begin{pmatrix} x_0 \\ y_0 \\ z_0 \end{pmatrix} - \begin{pmatrix} x_0(Po) \\ y_0(Po) \\ z_0(OrC) \end{pmatrix}$$

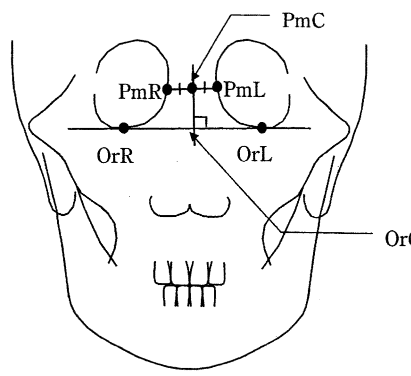


FIG 5 Decision of OrC.

Second processing: method B sequential correction for cephalic revolution, which is divided into elements of x-, y-, *and* z-*axes.* The method corrects for cephalic revolution around x- and y-axes sequentially, in addition to around the z-axis. After correcting for cephalic revolution around the z-axis as described in method A, the following calculation is carried out.

1. Correction for cephalic revolution around the x-*axis.* The revolution angle around the x-axis, θ_F (Figure 6 anticlockwise direction as positive), is the angle between the straight line passing bilateral Or and the z-axis on the frontal cephalogram. It is calculated by the following numerical formula:

$$\theta_F = \text{Arctan}[y_{PA}(\text{OrR})\text{-}y_{PA}(\text{OrL})/z_{PA}(\text{OrL}) - z_{PA}(\text{OrR})]$$

The co-ordinate (x', y', z'), after correcting for revolution, is calculated by the following numerical formula:

$$\begin{pmatrix} x' \\ y' \\ z' \end{pmatrix} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & \cos\theta_F & \sin\theta_F \\ 0 & -\sin\theta_F & \cos\theta_F \end{pmatrix} \begin{pmatrix} x_0 \\ y_0 \\ z_0 \end{pmatrix}$$

2. Correcting cephalic revolution around the y-*axis.* The revolution angle around the y-axis, θ_H (Figure 7 anticlockwise direction as positive), is the angle between the straight line passing PoC and OrC, which is projected in the x–z plane and the x-axis. It is described by the following numerical formula:

$$\theta_H = \text{Arctan}\,[z_0(\text{OrC}) - z_0(\text{PoC})]/[x_0(\text{OrC}) - x_0(\text{PoC})]$$

In addition, as Po cannot be detected on the frontal cephalograms when measuring human subjects, the midpoint of the bilateral *processus mastoideus* is detected as a substitute point for z_{PA} (PoC). This point is substituted into numerical formula 7 and z_0 (PoC) is calculated. Moreover, z_0 (OrC) is calculated in the same way as method A.

The co-ordinate (x'', y''', z'') after correcting revolution is described by the following numerical formula:

$$\begin{pmatrix} x'' \\ y'' \\ z'' \end{pmatrix} = \begin{pmatrix} \cos\theta_H & 0 & \sin\theta_H \\ 0 & 1 & 0 \\ -\sin\theta_H & 0 & \cos\theta_H \end{pmatrix} \begin{pmatrix} x' \\ y' \\ z' \end{pmatrix}$$

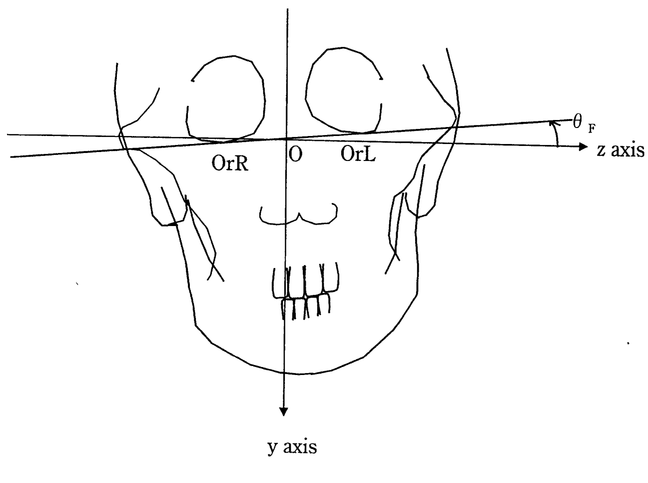


FIG 6 Cephalic revolution angle around the x axis (θ_F). θ_F is the angle between the z axis of the cephalogram coordinate system and the straight line which passes through RrR and OrL on the frontal cephalogram.

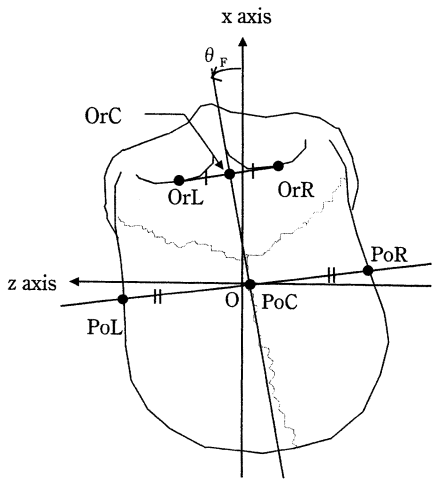


FIG 7 Cephalic revolution angle around the y axis (θ_H). θ_H is the angle between the x axis of the cephalogram coordinate system and the straight line passing PoC and OrC which is projected in the x–z plane.

3. Parallel translation of the origin. The co-ordinate (X_0, Y_0, Z_0) of point A on the standard co-ordinate system is calculated by the following numerical formula:

$$\begin{pmatrix} X_0 \\ Y_0 \\ Z_0 \end{pmatrix} = \begin{pmatrix} x'' \\ y'' \\ z'' \end{pmatrix} - \begin{pmatrix} x_0\,(\text{PoC}) \\ y_0\,(\text{PoC}) \\ z_0\,(\text{PoC}) \end{pmatrix}$$

Examination of accuracy

Examination of the influence of cephalic revolution on measurement accuracy. Lead 'leaves' were affixed to 12 landmarks on a human dry skull (Table 1). Measurement of the cephalogram of the human dry skull, which was exposed at positions revolved around x-, y-, and z-axes, was performed. The 3D co-ordinate of each landmark obtained with our system was compared with the true value obtained with the 3D digitizer (Tristation 400CNC, Nikon Co, Tokyo, Japan). Spatial accuracy within 5 μm was recorded. The accuracy of method A and B was compared.

Further examination of measurement accuracy

1. Calculation of the standard error. 'Error' is defined as the difference between a true value and a measured value. It includes both the systematic error and the random error. The systematic error is defined as a difference between the true value and the mean value. It is caused by the measurement system itself, such as characteristics of the cephalogram image, inputting of traced images into the image processor as mediated by a camera, the pixel size of the monitor, calculations in co-ordinates in the computer, and so on. A random error is unevenness in a measurement and it caused by human errors, such as tracing of the cephalogram, detection of landmarks, and so on. When accuracy is examined, both types of errors need to be considered.

Lead 'leaves' were affixed to twelve landmarks on seven human dry skulls (Table 1). The 3D co-ordinate of each landmark obtained with our system was compared with the true value. The standard error was calculated, as the linear sum of the systematic error and the random error, the confidence coefficient being 0·84.

$$\text{Standard error} = u/\sqrt{n} + \delta$$

where u = square root of unbiased variance, n = number of measurements, $u/\sqrt{n}$ = random error, and δ = systematic error.

Results

Construction of the 3D cephalometric system

A 3D cephalometric system was constructed. Magnification of the image and 3D cephalic malpositioning during cephalogram exposure were corrected and 3D co-ordinates of landmarks were calculated.

Accuracy

The influence of cephalic revolution on measurement accuracy. Measurement of cephalograms of human dry skulls, which were exposed at positions revolved around *x*-, *y*-, and *z*-axes, were taken and the differences from true values calculated. The accuracy of methods A and B was compared (Figure 8).

In the measurement of OrR, for example, when the skull was revolved around the *x*-axis (–5, –3, 0, 3, and 5 degrees), the difference from the true value in the *y* co-ordinate using method A increased with the degree of revolution and became about 3 mm at 5 degrees revolution. Moreover, when tendencies among all landmarks were investigated, difference in *y* and *z* co-ordinates increased with the degree of revolution when method A was applied. However, when method B was applied, differences were within 0·8 mm for all co-ordinates.

When the skull was revolved around the *y*-axis (–5, –3, 0, 3, and 5 degrees), the difference from the true value in the *x* co-ordinate using method A increased with the degree of revolution, and became about 3 mm at 5 degrees revolution. Moreover, when tendencies among all landmarks were investigated, the accuracy of measurement of *x* and *z* co-ordinates of bilateral Or, Po, and Go, apart from the midsagittal plane, decreased with method A. However, when method B was applied, differences were within 0·7 mm for all co-ordinates.

When the skull was revolved around the *z*-axis –10_10 degrees (5-degree increments), differences from true values were within 0·8 mm for all co-ordinates with both methods.

Although the tendency differed with the various landmarks, the accuracy of method A was markedly influenced by cephalic revolution. However, the accuracy of method B was not influenced by cephalic revolution.

Further examination of measurement accuracy. Further examination of the accuracy of method B was subsequently performed. The standard error was within 0·8 mm for the *x* co-ordinate, within 0·5 mm for the *y* co-ordinate, and within 0·8 mm for the *z* co-ordinate (Table 2).

TABLE 2 *Standard error of the system*

Landmark	Standard error (mm)		
	x	y	z
PoR	0·7	0·3	0·5
PoL	0·8	0·1	0·4
OrR	0·2	0·1	0·3
OrL	0·7	0·1	0·2
N	0·3	0·3	0·2
ANS	0·2	0·2	0·3
A	0·2	0·4	0·3
B	0·5	0·2	0·2
Pog	0·5	0·2	0·4
Me	0·6	0·2	0·2
GoR	0·7	0·5	0·8
GoL	0·5	0·5	0·8

Discussion

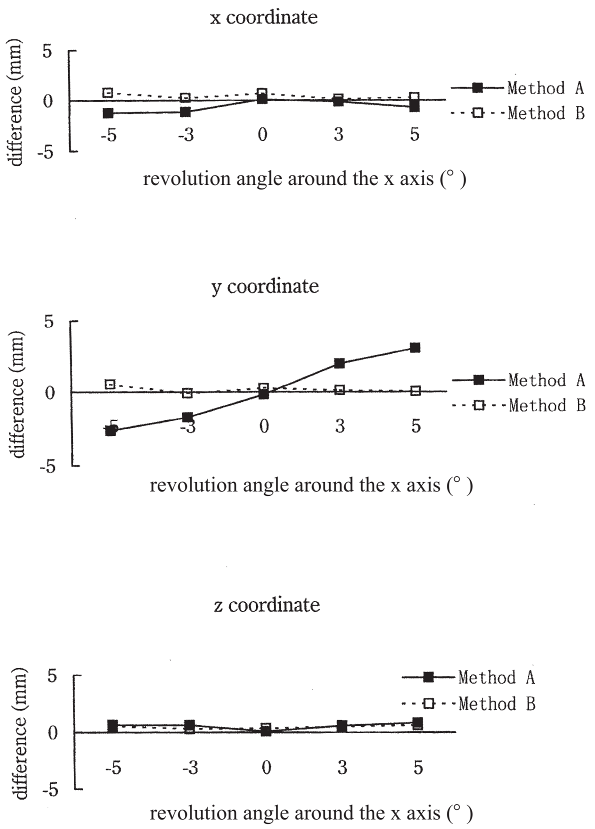
Since the introduction of cephalograms by Broadbent in 1931, various analytical methods have been established, most usually for lateral cephalograms. In recent years, methods that calculate 3D co-ordinates of orthodontic landmarks from lateral and frontal cephalograms have been developed. Since cephalic positioning is not usually accurate during cephalogram exposure, precise measurement requires correction for magnification of the image caused by the extension of X-ray bundles and 3D cephalic revolution.

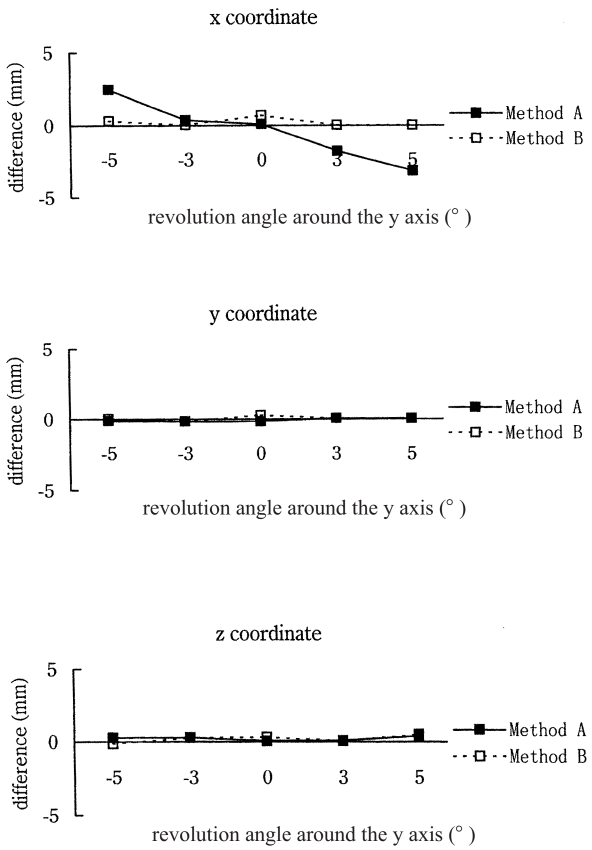
A new methodology for the correction of measurement errors caused by 3D cephalic revolution (method B) has been developed and its measurement accuracy examined. Cephalograms of a human dry skull positioned to revolve around *x*-, *y*-, and *z*-axes were measured. Measurement accuracy was unaffected by cephalic revolution around any of the axes. Further examination of the accuracy was performed by evaluating standard errors in the measurement of human dry skulls. Seven human dry skulls were used and each subject was exposed seven times. The data obtained can be used for statistical procedures in accordance with a normal distribution based on the central limit theorem.

The error in this system is attributed to following factors: the error associated with tracing and manual inputting of landmarks, and distortion of the image when traced images is input into the image processor using a camera and displayed on a monitor. Although our system is considered to possess sufficient accuracy for clinical applications, it necessitates certain changes in the hardware, an image scanner substituted for the camera, an image processor and a monitor with smaller pixels, and the development of software capable of detecting landmarks automatically in order to improve measurement accuracy.

Conclusions

A 3D cephalometric system, which corrects magnification of the image and cephalic malpositioning in cephalograms three-dimensionally, was developed to improve the accuracy of cephalometric measurements for the purpose of clinical application. By this method, magnification of the image was corrected for first and cephalic revolution was subsequently sequentially corrected. In order to examine the accuracy





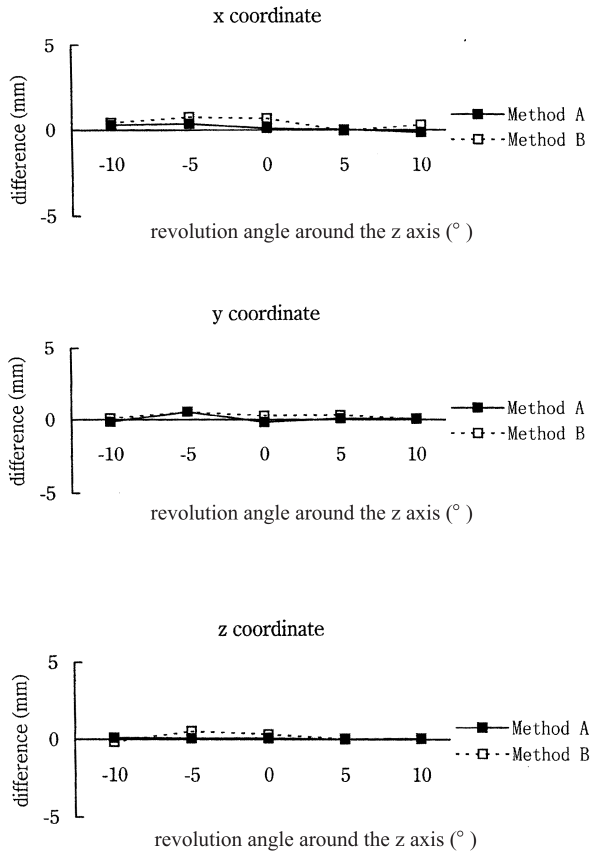


FIG. 8a Influence of cephalic revolution on measurement accuracy. Three-dimensional coordinate of OrR at revolving the dry skull around the x axis was obtained with our system. Difference between measured value and the true value is presented.

FIG. 8b Influence of cephalic revolution on measurement accuracy. Result of same examination at revolving the dry skull around the y axis.

FIG. 8c Influence of cephalic revolution on measurement accuracy. Result of same examination at revolving the dry skull around the z axis.

of this system, measurements were made of seven human dry skulls. Results indicated that precise cephalometric measurement became possible with this new system and it was suggested its clinical application would be possible.

References

Bookstein F. L., Grayson B. H., Cutting C. B., Kim H. and McCarthy J. G. (1991)
Landmarks in three dimensions: reconstruction from cephalograms versus direct observation,
American Journal of Orthodontics and Dentofacial Orthopedics, **100(2),** 133–132.

Broadbent B. H. (1931)
A new X-ray technique and its application to orthodontia,
Angle Orthodontist, **1,** 45–66.

Downs W. B. (1948)
Variations in facial relationship: their significance in treatment and prognosis,
American Journal of Orthodontics, **34,** 812–840.

Graber T. M. (1952)
New horizons in case analysis—clinical cephalometrics,
American Journal of Orthodontics, **38,** 603–624.

Grayson B. H., Cutting C. B., Bookstein F. L., Kim H. and McCarthy J. G. (1988)
The three-dimensional cephalogram: theory, technique, and clinical application,
American Journal of Orthodontics and Dentofacial Orthopedics, **94(4),** 327–337.

Ricketts R. M. (1966)
Cephalometric synthesis,
American Journal of Orthodontics, **46,** 647–673.

Savara B. S. (1965)
A method for measuring facial bone growth in three dimensions,
Human Biology, **37,** 245–255.

Spolyar J. L., Vasileff W. and Macintosh R. B. (1993)
Image corrected cephalometric analysis (ICCA): design and evaluation,
Cleft Palate-Craniofacial Journal, **30(6),** 528–539.

Tasman B. and Amanda H. A. (1989)
Computer assisted location of reference points in three dimensions for radiographic cephalometry,
American Journal of Orthodontics and Dentofacial Orthopedics, **95(6),** 490–498.

Tweed C. H. (1946)
The Frankfort-mandibular plane angle in orthodontics,
American Journal of Orthodontics, **32,** 175–230.

Yamazaki T. and Iwasawa T. (1981)
A study on the three-dimensional position and morphology of the mandible in cephalic region—with reference to comparison between the normal occlusion and true mandiblar protrusion groups,
Journal of Japan Orthodontic Society, **40(2),** 205–227. [In Japanese]